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HEAT TRANSFER AND HEAT CONDUCTION IN TECHNOLOGICAL PROCESSES

SIMULATION OF A MANUAL ELECTRIC-ARC WELDING IN A WORKING GAS PIPELINE. 1. FORMULATION OF THE PROBLEM

V. I. Baikov, I. A. Gishkelyuk, A. M. Rus', T. V. Sidorovich, and B. A. Tonkonogov

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Problems of mathematical simulation of the temperature stresses arising in the wall of a pipe of a cross-country gas pipeline in the process of electric-arc welding of defects in it have been considered. Mathematical models of formation of temperatures, deformations, and stresses in a gas pipe subjected to phase transformations have been developed. These models were numerically realized in the form of algorithms representing a part of an application-program package. Results of verification of the computational complex and calculation results obtained with it are presented.

Keywords: *cross-country gas pipeline, metal, phase composition, electric-arc welding, mathematical simulation, algorithm, numerical methods, temperature stresses.*

Introduction. Among the defects arising in the pipes of cross-country gas pipelines in the process of their work are cavities of different depth and longitudinal cracks. To eliminate these defects, a manual electric-arc welding or facings are used [1, 2].

The quality of a welded joint in a gas pipe is mainly determined by the welding parameters. The distributions of temperatures, phases, and stresses in the wall of a gas pipe subjected to welding characterize the welding process to full measure and allow one to determine its optimum parameters. Determination of the thermal-action zone and the residual stresses in such a pipe is a fairly complex problem, for solution of which it is advantageous to use mathematical models allowing one to calculate the distributions of temperatures, phases, and stresses in the wall of the indicated gas pipe in the dynamic regime. The control of these parameters of a gas pipe subjected to electric-arc welding makes it possible to realize optimum conditions of its welding in practice.

Mathematical Model. We consider the main problems of mathematical simulation of the physical processes arising in the wall of a gas pipe as a result of the elimination of a nonthrough crack in it by electric-arc welding [2].

A surface region of a gas pipe, containing a cavity of length L extending along the X axis in the three-dimensional Cartesian coordinate system XYZ is considered. The thickness of the pipe wall is equal to H. The pipe has a large diameter $D \sim 1$ m; therefore, the indicated surface region can be considered as a plane. The problem being investigated is symmetrical, which allows one to perform calculations for a half-pipe region (Fig. 1).

An electric-arc heat source (a welding electrode) moves along the cavity. The conditions of heating and penetration of the plate to a definite depth can be realized at definite values of the voltage (potential difference) U across the electrode and the tube, the strength of the current experienced by the arc, the area of the electrode spot on the surface of the pipe S_{el}, and the velocity of movement of the electrode v_{el}. In the process of welding of the pipe, the temperature field penetrates into its wall; therefore, the welding conditions must provide an inner-surface temperature that would be explosion-proof for the gas pumped through the pipe.

A mathematical description of the heat transfer in the indicated plate is based on the heat-conduction equation

A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15 P. Brovka Str., Minsk, 220072, Belarus; email: lusid@hmti.ac.by. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 83, No. 5, pp. 954–964, September–October, 2010. Original article submitted November 9, 2009.

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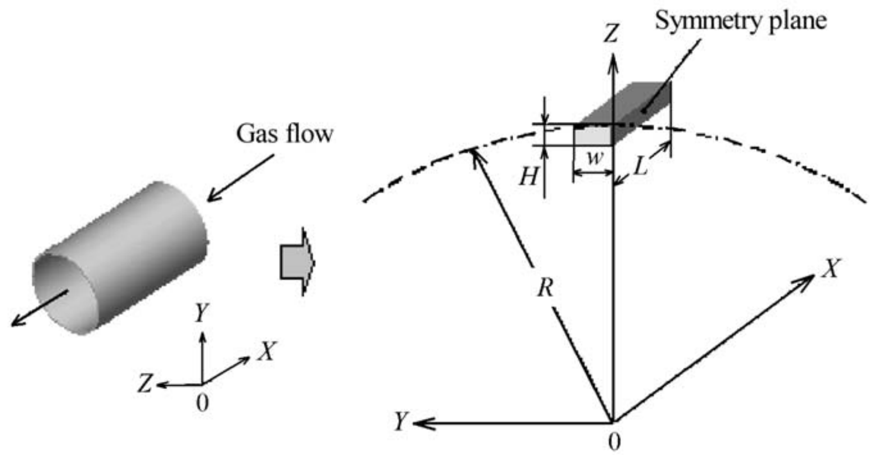

Fig. 1. Graphic representation of separation of a computational region for mathematical simulation of the formation of temperatures, deformations, and stresses in the wall of a gas pipe.

$$C_v \frac{\partial T}{\partial t} = \mathrm{div}\,(\lambda\, \mathrm{grad}\,(T))\,. \tag{1}$$

The energy afflux to the outer surface of the pipe is determined by the power of the electric-arc heat source:

$$q_{\mathrm{el}} = \frac{AIU}{S_{\mathrm{el}}} \exp\left(-k\left[(x - v_{\mathrm{el}}t)^2 + y^2\right]\right). \tag{2}$$

The heat losses in the environment and the energy expended for the melting-down of the electrode and the welding wire can be estimated with the use of the coefficient of utilization of the arc power A, which is usually equal to 0.5–0.7. The Gaussian shape of the coordinate dependence points to the fact that the arc has a hot core and a relatively cold periphery; the expression $x - v_{\mathrm{el}}t$ defines the movement of the heat source (the electric arc) along the X axis with a velocity v_{el}. The region S_{el} is bounded by the quantities $q_{\mathrm{el}}(r) \geq 0.005 q_{\max}(r)$, where $r^2 = (x - v_{\mathrm{el}}t)^2 + y^2$.

The heat transfer from the surface of the pipe to the environment is estimated by the law of convective heat exchange with the environment:

$$q = \alpha\,(T - T_{\mathrm{env}})\,. \tag{3}$$

A detailed mathematical description of the formation of deformations and stresses in a gas pipe in the process of its welding is presented in [1, 2]. The appearance of these defects is due to the nonuniform linear thermal expansion of the material of the pipe, the formation of different-density phases in it, and the gas pressure inside the pipe. The stresses arising in the material of a gas pipe as a result of its welding can substantially exceed the plastic limit of this material; therefore, both the elastic and plastic deformations should be taken into account in the calculations. Moreover, the problem is complicated by the fact that the deformations in the melted region should be calculated with account for the fact that this region is not a continuous solid body but a body containing both the solid and liquid phases [3–5]. The stresses in the elastic region are calculated on the basis of Hooke's law relating the deformations with the stresses [6, 7], and the stresses in the plastic region are calculated with the use of the associative law of a plastic flow [8]. The transformation of the body of the indicated pipe from the elastic state into the plastic one is characterized by the Mises yield function, which is used widely for calculating the deformations and stresses in steel bodies [6, 8–10]. A deformation of the body is determined by the changes in its temperature and phase composition, causing the volume of the body to change [6]. The mechanical properties — the Young modulus, the Poisson ratio, the yield point, the modulus of elasticity, and the coefficient of thermal linear expansion — of the body are determined by its phase composition and temperature.

We now consider the main equations used for calculating the heat stresses and deformations in the wall of a gas pipe subjected to electric-arc welding. The model for calculating the deformations and stresses arising in the material of this pipe is based on the equilibrium condition [6, 9]

$$-\nabla\cdot\boldsymbol{\sigma} = \mathbf{f}\,. \tag{4}$$

The deformations of the body of the indicated pipe are caused by the changes in its temperature and phase composition [9]:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_0 + \boldsymbol{\varepsilon}_{\text{th}} + \boldsymbol{\varepsilon}_{\text{ph}} + \boldsymbol{\varepsilon}_{\text{ep}}\,. \tag{5}$$

The stresses arising in the material of the pipe are expressed in terms of deformations [6, 8, 10]:

$$\boldsymbol{\sigma} = \mathbf{D}\,(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0 - \boldsymbol{\varepsilon}_{\text{th}} - \boldsymbol{\varepsilon}_{\text{ph}}) + \boldsymbol{\sigma}_0\,. \tag{6}$$

For small displacements taking place in the process of welding, the relation between the deformations and the displacements is determined by the expressions

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}\,,\ \ \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial \upsilon}{\partial x}\right),\ \ \varepsilon_{yy} = \frac{\partial \upsilon}{\partial y}\,,\ \ \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial \upsilon}{\partial z} + \frac{\partial \omega}{\partial y}\right),\ \ \varepsilon_{zz} = \frac{\partial \omega}{\partial z}\,,\ \ \varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial \omega}{\partial x} + \frac{\partial u}{\partial z}\right). \tag{7}$$

The transformation of the pipe material from the elastic state into the plastic one is defined by the von Mises yield function

$$f(\sigma) = \overline{\sigma} - \sigma_{\text{th}}\,. \tag{8}$$

The value of the stress arising in this case is determined by the formula

$$\overline{\sigma} = \sqrt{\frac{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6\,(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2)}{2}}\,. \tag{9}$$

In the plastic region, the relation between the stresses and the deformations is defined as [8]

$$\boldsymbol{\sigma} = \mathbf{D}_{\text{ep}}\,(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0 - \boldsymbol{\varepsilon}_{\text{th}} - \boldsymbol{\varepsilon}_{\text{ph}})\,. \tag{10}$$

Substitution of the dependences of the stresses on the deformations, determined by (6), and the dependences of the deformations on the displacements, determined by (7), into the equilibrium equation (4) gives a system of three nonlinear partial differential equations that can be written in the vector form

$$\nabla\,(\mathbf{c}\,(u)\cdot\nabla\mathbf{u} + \beta T + \delta) = \mathbf{f}\,. \tag{11}$$

Numerical Solution of the Differential Equations Obtained by the Finite-Element Method. The equations describing the distributions of temperatures and stresses in the wall of a gas pipe with corresponding boundary and initial conditions are solved by numerical methods [1, 8, 11–14].

The authors' experience suggests that, in the problem being considered, it is appropriate to use the finite-element method because it allows one to relatively easily perform a geometric discretization of the model and adequately simulate the effects described by the partial differential equations. In this case, to increase the accuracy and reliability of the calculations it is necessary to take into account the phase changes arising in the material of a gas pipe subjected to electric-arc welding [1, 10–15] and the dependence of the thermal and mechanical properties of this material on its temperature and phase composition. For a grid discretization of the model, it is advisable to use tetrahedral finite elements, which will make it possible to fairly accurately describe the geometry of different objects.

The block diagram of the computer realization of the mathematical model of the process of electric-arc welding of a gas pipe is presented in Fig. 2.

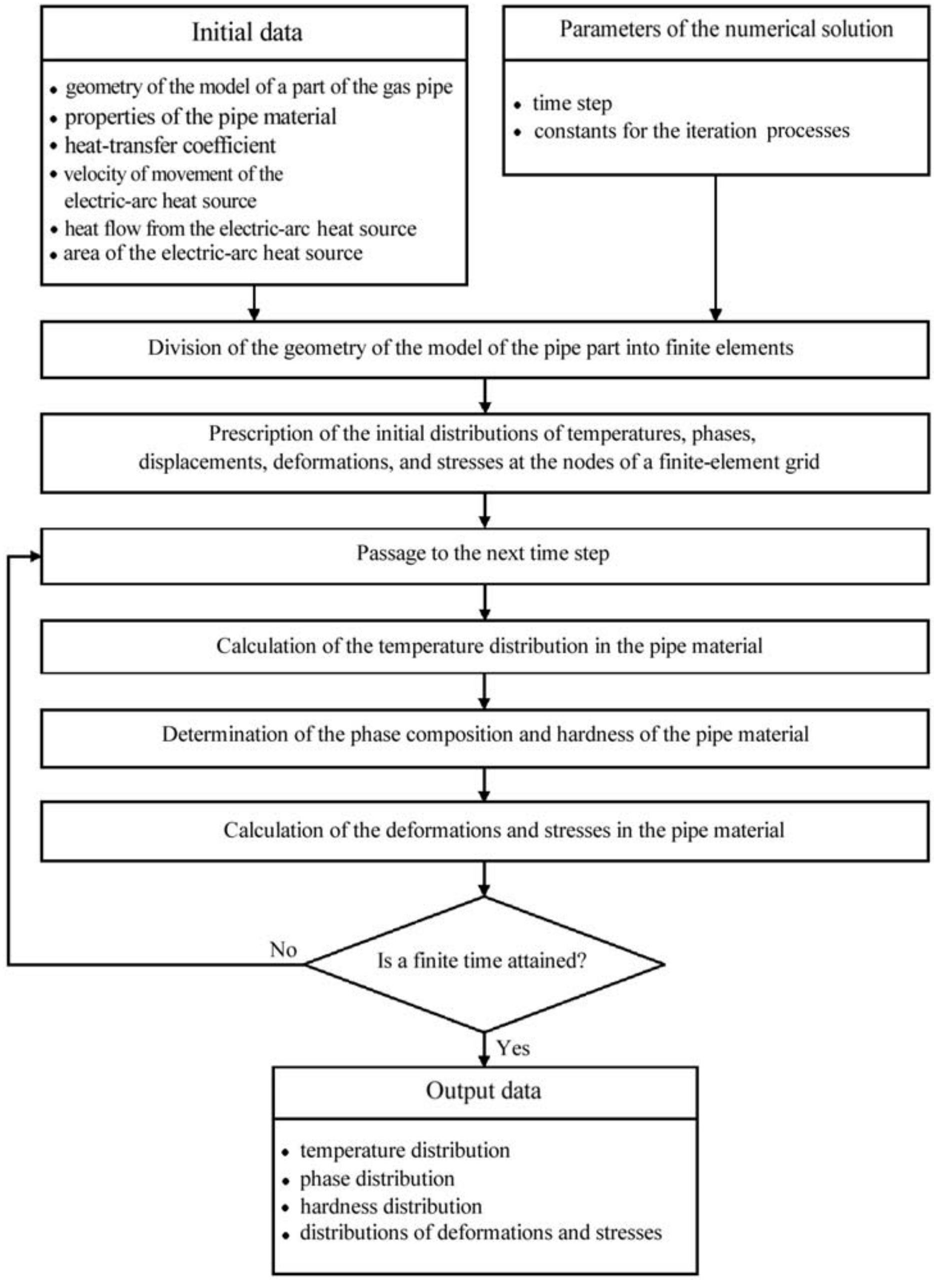

Fig. 2. Logical scheme of complex simulation of the thermophysical processes in a gas pipe subjected to electric-arc welding.

Thus, an algorithm for numerically solving the heat problem with account for the phase transformations of the material of a gas pipe subjected to electric-arc welding has been realized. The heat processes occurring in this pipe were described with the use of the nonstationary heat-conduction equation (the Fourier–Kirchhoff equation) with corresponding boundary conditions accounting for the convective and radiative heat exchange. The results of solution of this equation were discussed in a number of works and were widely used in commercial program complexes [16–18]. However, in these works, the heat fields arising in the material of the indicated pipe in the process of its heating and cooling were mathematically described without considering their dependence on the phase transformations of the pipe material and the changes in its properties depending on the phase composition of the material and the heat absorbed or released by it as a result of its phase transformations, which was emphasized in the present work.

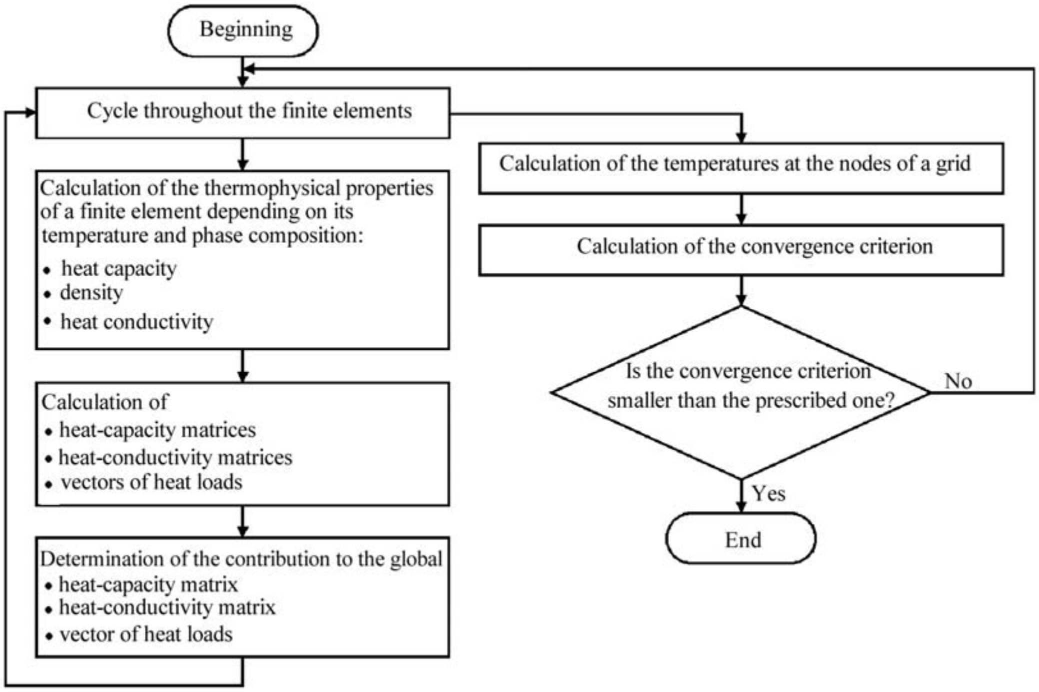

Fig. 3. Logical scheme for solving the heat problem.

The temperature distribution in the material of a gas pipe subjected to electric-arc welding is determined from the solution of the heat-conduction equation with boundary and initial conditions by the finite-element method [19–21]. In this case, the heat-conduction equation is reduced to the system of equations

$$\frac{\mathbf{C}\,(\mathbf{T}^{n+\theta})}{\Delta t}\,(-\,\mathbf{T}^{n}+\mathbf{T}^{n+1})+\mathbf{P}\,(\mathbf{T}^{n+\theta})\,\left[(1-\theta)\cdot\mathbf{T}^{n}+\theta\cdot\mathbf{T}^{n+1}\right]=\mathbf{F}^{n+\theta}\,. \tag{12}$$

Here $\mathbf{T}^{n+\theta}$ and $\mathbf{F}^{n+\theta}$ are the temperature and the heat load calculated at the instant of time $t = t + \theta\Delta t$, where $0 \le \theta \le 1$, as the following approximations:

$$\begin{aligned} \mathbf{T}^{n+\theta} &= (1-\theta)\cdot\mathbf{T}^{n}+\theta\cdot\mathbf{T}^{n+1}\,, \\ \mathbf{F}^{n+\theta} &= (1-\theta)\cdot\mathbf{F}^{n}+\theta\cdot\mathbf{F}^{n+1}\,. \end{aligned} \tag{13}$$

This formation was made with the use of the known approach [19, 21]. However, in the heat-conduction equation being used, unlike the classical nonlinear formulation, the fact that the heat capacity, density, and heat conductivity of a steel depend not only on its temperature but also on the phase composition and that the density of the inner heat sources of the steel is determined by the absorption and release of energy as a result of its phase transformations is additionally taken into account. This allows one to simulate the formation of heat fields in the material of a gas pipe subjected to electric-arc welding and to solve the problem on their propagation with account for the phase transformations of this material, substantially influencing its thermal properties. For example, the enthalpy of the pipe material, determined by the release or absorption of latent heat as a result of the reconstruction of its crystal lattice, substantially changes in the process of phase transformations of this material; in this case, the heat conductivity of the material changes too because different phases possess different heat conductivities.

To determine the temperature $\mathbf{T}^{n+1}$, the system of equations (12) is solved iteratively in each time step. In the first iteration, the values of $\mathbf{T}^{n+\theta}$ and $\mathbf{F}^{n+\theta}$ are calculated from expressions (13). As a result of the solution of the system of equations (12), the first approximation of $\mathbf{T}^{n+1}$ is obtained. In the next iterations the values of $\mathbf{T}^{n+\theta}$ and

$\mathbf{F}^{n+\theta}$ are determined by formulas (13). The iteration is performed as long as the difference between the temperatures at two adjacent increments exceeds the error $\xi = \left(\frac{\mathbf{T}^{n+1} - \mathbf{T}^{n}}{\mathbf{T}^{n+1}}\right)^2$. Thus, the iterative solution of the system of equations (12) in the current time step gives the values of the temperatures at all the nodes of the finite-element grid at the instant of time $t = t + \Delta t$. These temperatures are used for determining the new phase composition of the pipe material and calculating its stressed-strained state. Then the temperatures in the next time step are determined, and the procedure is performed as long as a given time is attained.

The logical scheme of solving the heat problem shown in Fig. 3 was constructed on the basis of the above-described model. A peculiarity of this algorithm is that it accounts for the dependence of the properties of the material of a gas pipe subjected to electric-arc welding not only on its temperature but also on the phase composition of the pipe material and the heat released or absorbed by it as a result of its phase transformations. In this case, a set of data on the dependence of the thermophysical properties of each finite element of the material on its phase composition at the beginning of the calculation (initiated with initial conditions) or on the phase composition calculated as a function of the rate of change in the temperature of the material in the process of its heating or cooling is formed, and each phase is characterized by the dependence of the thermal properties of the material on its temperature. These data are transmitted algorithmically for calculating the local and global matrices and vectors of the parameters of the material of the gas pipe and, as a result, for solving the problem on the distribution of heat fields in the pipe material with account for its phase transformations by the finite-element method.

The material of a gas pipe can experience the following structural transformations in the process of its welding:

a) the formation of austenite as a result of the heating of the material;

b) the decomposition of the austenite into ferrite, pearlite, and bainite as a result of the cooling of the material;

c) the transformation of the austenite into martensite in the process of cooling of the material.

To calculate the kinetics of the phase transformations of the material of a gas pipe subjected to electric-arc welding (its phase composition) in the current increment, a model based on the approximation of the thermokinetic or isothermal diagrams of formation or decomposition of the overcooled austenite was proposed in [22]. As a rule, the thermokinetic diagrams presented in domestic reference books carry more complete information than the isothermal diagrams; however, in some cases, the latter give more correct information and are better described algorithmically with the use of the Avrami diffusion equation [1].

The initial data for the use of isothermal diagrams are as follows:

1) an isothermal diagram in the form of lines of the beginning and end of transformations, approximated with sufficient accuracy by a set of parabolic relations obtained with the use of the least-square technique;

2) the ultimate fraction of the phase being calculated;

3) the temperature dependence of the hardness of the phase being calculated.

The method based on the determination of the phase composition of the material of a gas pipe by approximation of its isothermal diagram consists of calculating the structural transformations of the material (the transformation of the austenite into ferrite, pearlite, and bainite) in the process of its arbitrary cooling with the use of the theory of isokinetic reactions. This theory is based on the additivity rule for passing from the isothermal kinetics of transformations, described conventionally by the Avrami equation, to the nonstationary temperature conditions. It is known from practice that the use of this method allows one to obtain the best agreement between the calculation and experimental data. The structural components of the material are calculated simultaneously with solution of the heat-conduction equation. Numerical calculations are carried out with the use of the step method, in which the coefficients for each temperature are determined by the isothermal diagram of decomposition of the overcooled austenite.

Thus, algorithms for determining the phase composition and hardness of the material of a gas-pipe region subjected to electric-arc welding with the use of its thermokinetic and isothermal diagrams have been developed. The algorithm for calculating the phase composition and hardness of this material on the basis of the approximation of its isothermal diagram is presented in Fig. 4.

The problem of calculating the temperature deformations and stresses arising in the wall of a pipe of a cross-country gas pipeline in the process of welding-up of defects in it is complicated by the fact that the temperature distribution in the body of this pipe changes with time. Therefore, when the stressed-strained state of such a pipe is

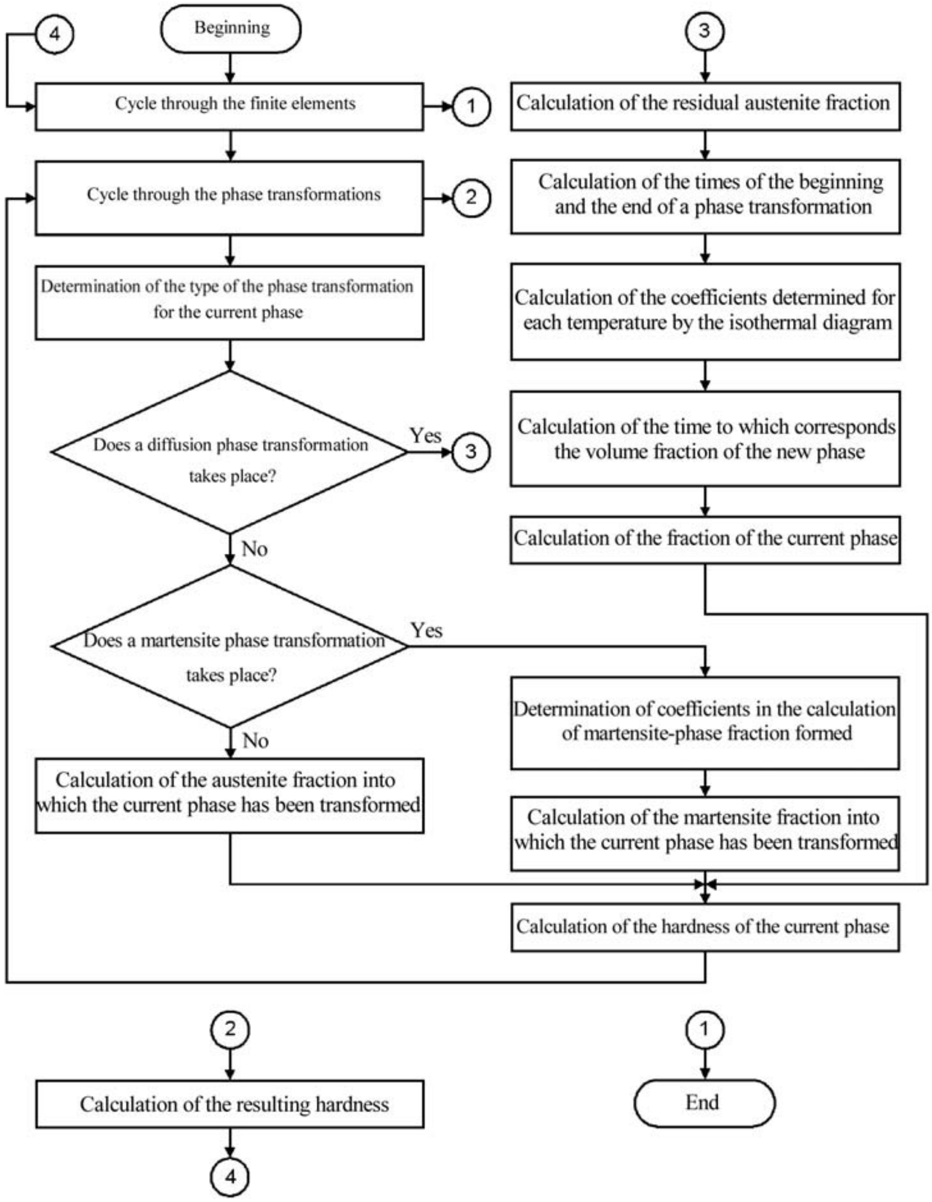

Fig. 4. Logical scheme for determining the phase composition and hardness of the material of a gas pipe subjected to electric-arc welding by approximation of its isothermal diagram.

analyzed, first the increase in the deformations and stresses arising in the previous increment of time is determined, and then the deformations and stresses accumulated in the pipe material are calculated. Moreover, the mechanical properties (the Young modulus, the Poisson ratio, the yield point, the modulus of elasticity, and the coefficient of thermal linear expansion) of a finite region of the pipe depend on its phase composition and temperature at a current time. These parameters are determined by analogy with calculation of the thermophysical properties of the pipe material as a function of its phase composition. A peculiarity of the method proposed for simulation of the distributions of deformations and stresses in the material of a gas pipe subjected to electric-arc welding as compared to the known methods is that it allows one to determine the dependence of the mechanical properties of the pipe material on its phase composition. For example, in the process of phase transformations of the indicated material, its elasticity and plasticity change significantly, because different phases possess different elastic and plastic properties, and the specific volume of the phases changes too, with the result that additional deformations and stresses arise in the material.

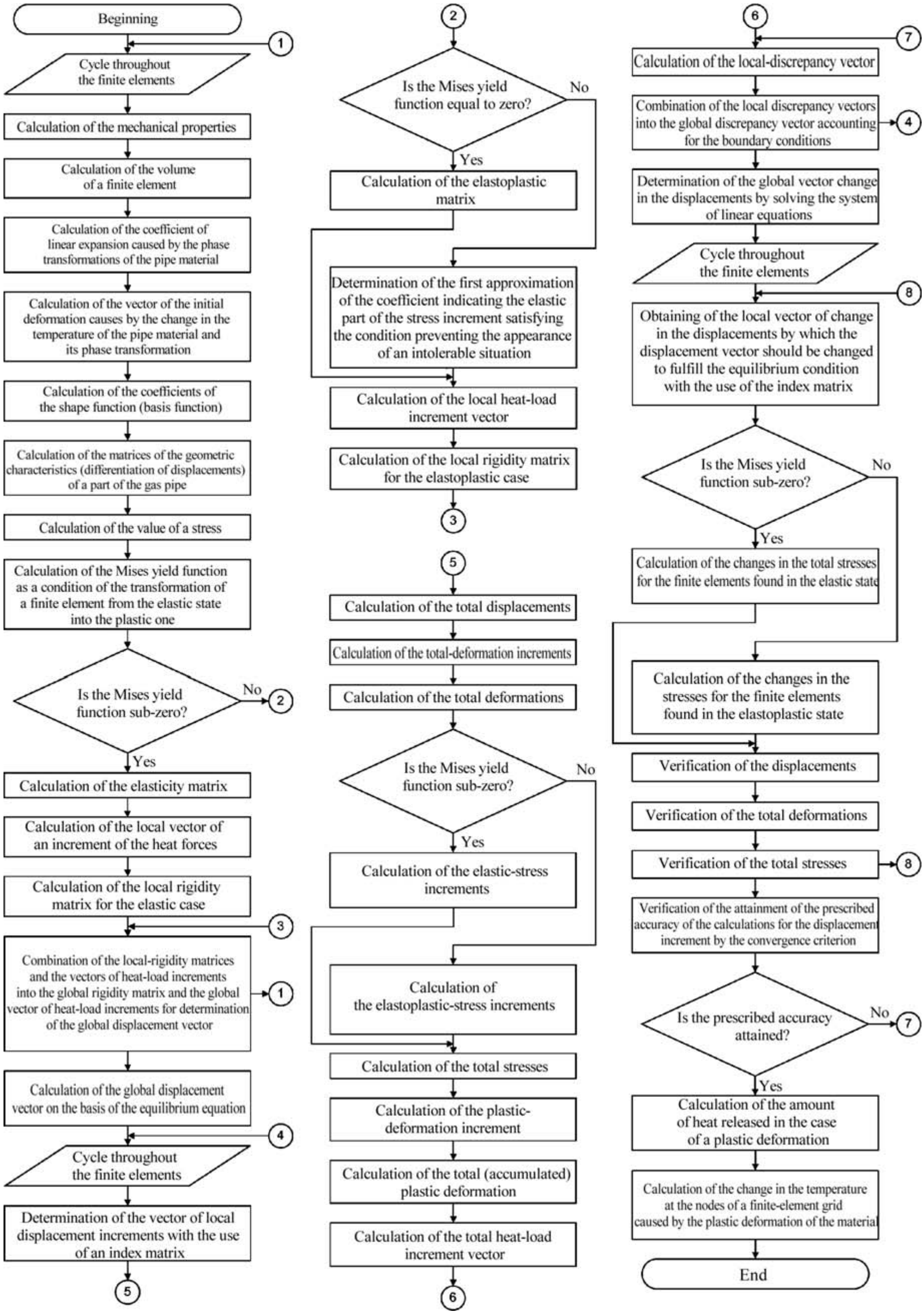

Fig. 5. Logical scheme of the algorithm for calculating the stressed-strained state of the material of a gas pipe subjected to electric-arc welding.

Thus, an algorithm for numerical realization of the problem on the stressed-strained state of the body of a gas pipe subjected to electric-arc welding (calculation of the displacements, deformations, and stresses at any point of it) with account for the phase transformations of each finite (tetrahedral) element of the pipe material (the changes in the mechanical properties of the pipe material depending on its phase composition) in one iteration cycle with the solution of the heat, phase-transformation, and elastoplastic problems has been developed. In this case, the final solution of the general problem is obtained when a finite temperature is attained (Fig. 5) [19, 20, 22].

A peculiarity of the algorithm proposed is that it, unlike the known approaches, allows one to calculate the dependence of the properties of a steel not only on its temperature but also on the phase composition of all its finite elements as well as organize a continuous cycle of transmission of data between the models of the stressed-strained state, the heat fields, the phase transformations, and the hardness of the steel. In this case, by analogy with the solution of the heat problem, additional data on the mechanical properties of each finite element of the steel, depending on its phase composition predetermined as initial conditions or calculated as a function of the rate of change in the temperature of the steel in the process of its heating or cooling, are obtained. These data are transmitted algorithmically to calculate the coefficient of linear expansion and different vectors and matrices of the steel parameters that are dependent on the change in the temperature of the steel and its phase composition and, as a result, calculate the increase in the displacements and in the other parameters of the stressed-strained state of the steel material by the finite-element method.

The mathematical models and algorithms developed for calculating the thermodynamic and stressed-strained states of the material of a gas pipe subjected to electric-arc welding with account for its phase transformations were realized as part of an adapted programmed module (processor). Special programs on the loading of files defining the geometry of a region of a gas pipe in a definite format; on the division of this geometry into finite elements; on the representation, rotation, and scaling of the elements and their interaction with the database on the properties of the pipe material; on the formation of the initial file of the project, and on the survey and analysis of the calculation results (representation of the distributions of temperatures, phases, deformations, stresses, and hardness over the volume of the model of the pipe region being considered by colored fields in definite time steps of the calculation and construction of graphic dependences of the above-indicated quantities on the welding time). In this case, for visualization of the fields of temperatures and stresses, the MATLAB program and its FEMLAB supplement were used.

Verification of the Mathematical Models. To verify the models developed we compared the calculation results with the data of a thermographic investigation carried out with the use of a special infrared imager. A three-dimensional region of a gas pipe was calculated with account for its symmetry (the visualization of the calculation data in the XY plane was carried out with the use of the MATLAB program package). As a result of the thermographic investigations, we obtained thermograms on the dynamics of the temperature fields in the material of the pipe in the process of welding-up of defects like cavities in it. However, a simple comparison of these fields gives no way of estimating the calculation error. This is explained by the fact that, in the IR-Preview program means (for visualization of experimental results) and the MATLAB program, different colored schemes and gradients are used for visualization of temperature fields. Therefore, graphs of changes in the temperature of the pipe material were constructed on the basis of experimental data with the use of the IR-Preview and Microsoft Excel programs for different profiles of the computational region (along the X and Y axes). These dependences were compared with calculation results. As is seen from Fig. 6, the calculation data agree well with the experimental ones: the average error in the calculation of the temperature distribution along the profile of the pipe region being considered in the temperature range 500–1100^{o}C did not exceed 8%; however, in the temperature range 200–500^{o}C it reached 20%. It should be noted also that the temperature range in which the infrared imager could make measurements was 145–1125^{o}C. It has been impossible to measure the temperatures that were higher or lower than the indicated ones; they were "cut," which is seen on the graph presented.

In our opinion, the finite size of the computational region is mainly responsible for the calculation error. It has been impossible to simulate the process for the whole pipe because of the complexity of the calculations; therefore, we considered a part of the pipe and assumed that the heat flow at the boundaries of the computational region is equal to zero. However, in reality, a conductive heat flow to the other parts of the pipe always takes place, which causes the pipe part being considered to cool more rapidly. Since it is very difficult to obtain experimental data on the

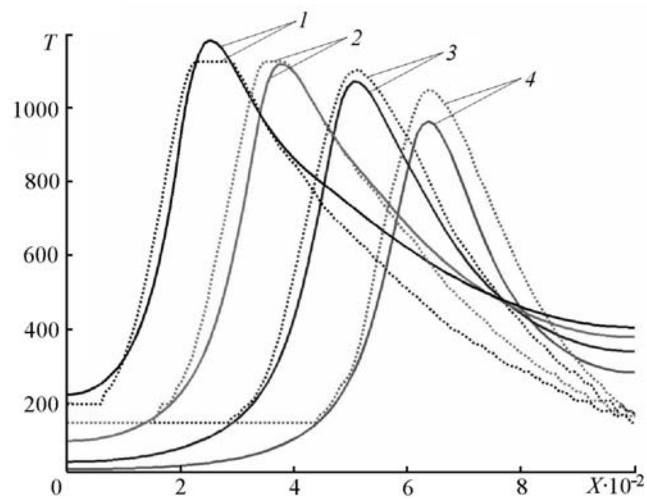

Fig. 6. Comparison of the calculated temperature distribution along the X axis (solid curves) with the corresponding experimental data (dotted lines) for the instants of time 50 (1), 40 (2), 30 (3), and 20 s (4).

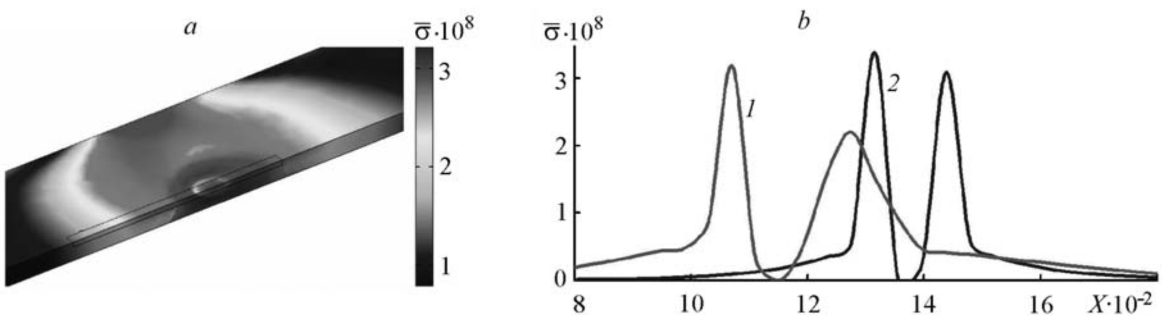

Fig. 7. Pattern of the stress distribution in the material of a gas pipe at the 20th second of its welding (a) and this distribution along the X axis of the computational region (b): 1) at the 20th second; 2) at the 2nd second.

change in the heat stresses arising in a gas pipe in the process of its welding, the model being considered was verified only for the temperature field formed in the pipe material.

A three-dimensional model with moving boundaries can be used for estimation of the gradual welding-up of a cavity in the wall of a gas pipe. In this model, the geometry of the computational region changes with the rate of welding-up of the defect region. However, this model is more complex and calls for a larger computational capacity.

As a result of the simulation of the stressed-strained state of part of a gas pipe, we obtained distributions of thermal deformations and stresses in this part (Fig. 7).

It is seen from Figs. 6 and 7 that, as expected, the stresses in the gas-pipe part being investigated are mainly due to the inhomogeneous temperature distribution in it and, in the regions where the temperature of the pipe material exceeds its melting temperature due to the heating of it by the electric-arc heat source, the stresses tend to zero. This is explained, first, by the sharp decrease in the Young modulus, the plastic limit, and the coefficient of strengthening of the material of the gas pipe with increase in its temperature and, second, by the complete relaxation of the mechanical stresses in it (they tend to zero) as a result of its transformation from the solid phase into the liquid one.

In the process of welding-up of defects in a region of a gas pipe, maximum stresses arise at the initial instant of time, which is explained by the existence of maximum temperature gradients. In the next time interval, the pipe region, the initial temperature of which is equal to the environmental temperature, is sharply heated to the melting temperature under the action of the electric-arc heat source. Then smaller temperature gradients will take place because the temperature of the pipe region adjacent to the region subjected to the welding will be higher than the environmental temperature because of the outflow of heat from the more heated region.

Conclusions. A mathematical model describing the distributions of temperature fields and stresses in the wall of a pipe of a cross-country gas pipeline subjected to electric-arc welding has been developed. The description of the temperature fields is based on the heat-conduction equation, and the calculation of the stresses and deformations is based on Hooke's law and the associated plastic-flow rule.

Algorithms for numerical solution of the equations obtained have been developed on the basis of the finite-element method and have been realized with the use of a computer. A peculiarity of the algorithms proposed for numerical simulation of the thermophysical processes arising in a pipe of a cross-country gas pipeline as a result of the welding-up of defects in it is that these algorithms allow one to take into account the dependence of the properties of the pipe material on its temperature and phase composition.

The models proposed make it possible to determine the dynamic changes in the distributions of temperature fields, phases, deformations, and stresses in the wall of the indicated gas pipe and can serve as an effective tool for determining the optimum parameters of electric-arc welding. A verification of these models has shown that, in this case, the error in the calculation of the temperature of the pipe material does not exceed 8% in the temperature range 500–1100^{o}C and 20% in the temperature range 200–500^{o}C.

NOTATION

A, coefficient of utilization of the arc power; $\mathbf{C}$, heat-capacity matrix; $\mathbf{c}(u)$, matrix of the coefficient characterizing the pipe-material properties; C_v, heat capacity per unit volume, J/(m^3·K); D, diameter of a gas pipe, m; $\mathbf{D}$, matrix identical to the elasticity matrix $\mathbf{D}_e$ in the case of an elastic deformation and to the elastoplastic matrix $\mathbf{D}_{ep}$ in the case of a plastic deformation; $\mathbf{f}$, vector of the external forces, N; $\mathbf{F}$, vector of the heat forces; $f(\sigma)$, Mises yield function; H, thickness of the pipe wall, m; I, strength of the current carried by the arc, A; k, concentration coefficient; L, length of a cavity, m; n, number of an increment; $\mathbf{P}$, heat-conduction matrix; q, heat flow from the surface of the pipe, W/m^2; q_{max}, power of an electric-arc heat source at the center of the pipe region subjected to welding, W/m^2; q_{el}, power of the electric-arc heat source per unit area, W/m^2; r, distance from the center of the electric-arc heat source, m; R, radius of the pipe, m; S_{el}, area of the spot of the electric-arc heat source, m^2; t, time of action of the electric-arc source on the pipe surface, s; $\mathbf{T}$, temperature vector at the nodes of a grid, K; T, temperature of the pipe surface, K; T_{env}, environmental temperature, K; $\mathbf{u}$, vector of displacements, m; u, displacement along the X axis, m; U, electrical voltage (potential difference) across the electrode and the pipe, V; v_{el}, velocity of movement of the electric-arc heat source (electrode), m/s; w, width of the cavity; x, distance along the X axis, m; y, distance along the Y axis, m; α, coefficient of convective heat transfer, W/(m^2·K); β, coefficient of thermal linear expansion, 1/K; δ, coefficient of volume expansion caused by the phase transformations; $\boldsymbol{\varepsilon}$, total-deformation vector; $\boldsymbol{\varepsilon}_0$ initial-deformation vector; $\boldsymbol{\varepsilon}_{th}$, thermal-deformation vector; $\boldsymbol{\varepsilon}_{ep}$, elastoplastic-deformation vector; $\boldsymbol{\varepsilon}_{ph}$, vector of the deformation caused by the phase transformations; ε_{xx}, deformation along the X axis; ε_{xy}, deformation in the XY plane; ε_{xz}, deformation in the XZ plane; ε_{yy}, deformation along the Y axis; ε_{yz}, deformation in the YZ plane; ε_{zz}, deformation along the Z axis; λ, heat conductivity, W/(m·K); θ, finite-difference approximation parameter; ξ, error for comparison of the difference between the temperatures of the two adjacent regions; $\boldsymbol{\sigma}$, stress vector, Pa; $\overline{\sigma}$, value of a stress, Pa; $\boldsymbol{\sigma}_0$, vector of the initial stresses, Pa; σ_{xx}, normal stress along the X axis, Pa; σ_{xy}, tangential stress in the XY plane, Pa; σ_{xz}, tangential stress in XZ plane, Pa; σ_{yy}, normal stress along the Y axis, Pa; σ_{yz}, tangential stress in the YZ plane, Pa; σ_{zz}, tangential normal stress along the Z axis, Pa; σ_y, yield stress, Pa; υ, displacement along the Y axis, m; ω, displacement along the Z axis, m. Subscripts: env, environment; el, electric arc; max, maximum; 0, initial; th, thermal; ep, elastoplastic; y, yield.

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